DELEGATE

BOOKLET

19IBAM03 (F2F)

Task 1

Marking Activity 1

Show that $\frac{\sqrt{12}-1}{2-\sqrt{3}}$ can be written as $4+3\sqrt{3}$

Show your working clearly. (4)

Mark Scheme

M1 Method to rationalise $\frac{(\sqrt{12}-1)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$

M1 correct expansion of brackets $\frac{2\sqrt{12}-2+\sqrt{12}\sqrt{3}-\sqrt{3}}{4-3}$

B1 $\sqrt{12}=2\sqrt{3}$(may be seen before expansion)

A1 answer from fully correct working with all steps seen

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Response A

that $\frac{\sqrt{12}-1}{2-\sqrt{3}}$ can be written as $4+3\sqrt{3}$

your working clearly.

$\frac{\sqrt{12}-1}{2-\sqrt{3}} = 4+3\sqrt{3}$

$\sqrt{12}-1 = 4+3\sqrt{3} \times (2-\sqrt{3})$

$\sqrt{12}-1 = 8-4\sqrt{3}+6\sqrt{3}-9$

$\sqrt{12} = -1+2\sqrt{3}$

$2\sqrt{3} = \cancel{-1}\cancel{+1}+2\sqrt{3}$

$2\sqrt{3} = 2\sqrt{3}$

Left side = Right side

$\therefore \frac{\sqrt{12}-1}{2-\sqrt{3}}$ can be written as $4+3\sqrt{3}$

$\frac{\sqrt{12}-1}{2-\sqrt{3}}$

$= \cancel{\frac{2\sqrt{3}-1}{2-\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}}$

$= \cancel{\frac{6-1}{2-1} = \frac{5}{1}}$

$\frac{\sqrt{4\times 3}-1}{2-\sqrt{3}}$

[illegible]

Response B

$\frac{\sqrt{12}-1}{2-\sqrt{3}}$

$= \frac{\sqrt{12}-1 \times (2+\sqrt{3})}{2-\sqrt{3} \times (2+\sqrt{3})}$

$= \frac{2\sqrt{12}+\sqrt{36}-2-\sqrt{3}}{2^2-3}$

$\rightarrow \frac{2\sqrt{12}+\sqrt{36}-2+\sqrt{3}}{1}$

$= \cancel{2\sqrt{4\times 3}+\sqrt{12}}$

$= 2\sqrt{4\times 3}+6-2-\sqrt{3}$

$= 4\sqrt{3}-\sqrt{3}+4$

$= \cancel{4}\,3\sqrt{3}+4$

$= 4+3\sqrt{3}$

Response C

$$\frac{\sqrt{12}-1}{2-\sqrt{3}} = \frac{\sqrt{4\times 3}-1}{2-\sqrt{3}} = \frac{2\sqrt{3}-1}{2-\sqrt{3}} \times 2+\sqrt{3} = \frac{3\sqrt{3}}{4-\sqrt{3}}$$

$$\cancel{(2\sqrt{3}-1)(2+\sqrt{3}) = 4\sqrt{3}-2+6-\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{4}$$

$$(2\sqrt{3}-1)(2+\sqrt{3})$$

$$= 4\sqrt{3}-2+6-\sqrt{3}$$

$$(2-\sqrt{3})(2+\sqrt{3})$$

$$= 4-\cancel{2\sqrt{3}}+\cancel{2\sqrt{3}}-\sqrt{3}$$

$$= 4-\sqrt{3}$$

(Total for Question 10 is 4 marl

Task 2

$$2x\log_3 x - 3\log_3 x - 4x\log_9 4 + 6\log_9 4 = \log_3\left(\frac{x}{4}\right)^{(2x-3)}$$

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Task 3

Marking Activity 2

2 Solve the equations

$$y = x^2 - 6x + 5$$

$$y + x = 11$$

(5)

Mark Scheme

2	$x^2 - 6x + 5 = 11 - x$	M1
	$x^2 - 5x - 6 \ (= 0)$ OR $y^2 - 17y + 60 \ (= 0)$	A1
	$(x-6)(x+1) \ (= 0)$ $(y-12)(y-5) \ (= 0)$	dM1
	$x = 6,\ y = 5$	A1
	$x = -1,\ y = 12$	A1 [5]

M1 Obtain an equation in one variable. Must be quadratic but no simplification needed
A1 Correct simplified 3 term quadratic equation, terms in any order
dM1 Solve their quadratic by any valid means (see "General Principles")
A1 Either (x, y) pair correct or both x values or both y values correct
A1 Second pair correct. It must be clear how the values are paired. (Horizontally as shown or vertically is sufficient.)

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Response 1

$y + x = 11$

$y = 11 - x$

$11 - x = x^2 - 6x + 5$

$x^2 - 6x + x + 5 - 11 = 0$ $\qquad -6$

$x^2 + 5x - 6 = 0$ $\qquad -1 \quad 6$

$(x-1)(x+6)$

$x = 1 \qquad x = -6$

if $x = 1$ $\qquad$ if $x = -6$

$y = 11 - 1$ $\qquad$ $y = 11 + 6$

$y = 10$ $\qquad$ $y = 17$

Response 2

$y = x^2 - 6x + 5$

$x^2 - 5x - x + 5$

$x(x-5) - 1(x-5)$

$y = (x-1)(x-5)$

$y + x = 11$

$x = 11 - y$

$y = (11-y)^2 - 6(11-y) + 5$

$= (11-y)(11-y) - 66 + 6y + 5$

$= 121 - 11y - 11y + y^2 - 66 + 6y + 5$

$= 60 - 16y + y^2$

$x - 1 = 0$ or $x - 5 = 0$

$x = 1$ or $x = 5$

substitute x to find y

$y + x = 11$

$y + 1 = 11$ or $y + 5 = 11$

$y = 11 - 1$ or $y = 11 - 5$

$= 10$ or $y = 6$

$x = 1,\ y = 10$

$x = 5,\ y = 6$

Task 4

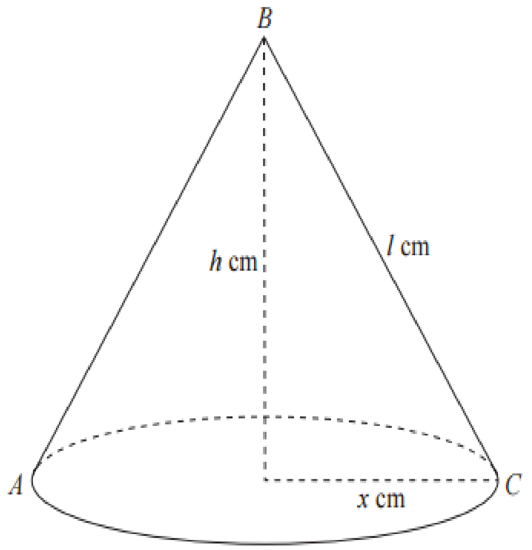

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a right circular cone with a base radius of x cm. The slant height of the cone is l cm and the height of the cone is h cm. The vertex of the cone is B and the points A and C, on the base of the cone, are such that AC is a diameter of the base.

The cone is increasing in size in such a way that the size of the angle ABC is constant at 60° and the **total** surface area of the cone is increasing at a constant rate of 10 cm^2/s.

Find the exact rate of increase of the volume of the cone when $x = 6$

(11)

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Scheme	Mark

Task 5
Marking Activity 3

5 In triangle ABC, $AB = 10$ cm, $BC = 7$ cm and angle $BAC = 40°$

(a) Find, in degrees to the nearest 0.1°, the two possible sizes of angle ACB.

(4)

(b) Find, in cm to 3 significant figures, the difference between the two possible lengths of AC.

(4)

Mark Scheme

		Notes
(a)	M1	For using a **correct** sine rule either way around
	A1	For rearranging sine rule to give $C = \sin^{-1}\left(\dfrac{10\sin 40}{7}\right)$ can be implied
	A1	$C = 66.7°$
	A1	$C = (180 - 66.7) = 113.3°$
(b)	M1	States or implies that triangle $CC'B$ is isosceles. This is all that is required for this mark.
	dM1	Use Cosine so length of half of base of triangle $BCC' = 7\cos(66.67)$
	A1	$CC' = 2 \times 7\cos(66.67)$ (follow through their **acute** angle ACB)
	A1	For 5.54 (cm)
	ALT	
	M1	For using an appropriate trig statement, ie., cosine rule or sine rule to find AC or AC'
	dM1	For a complete method to find the other length AC' or AC **and** the difference $(AC - AC' = \ldots)$
	A1	Correct length of AC or AC' (either awrt 10.4 or 4.89) (cm) This A mark is dependent on the first M mark only.
	A1	For 5.54 (cm)
		Total 8 marks

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Response 1

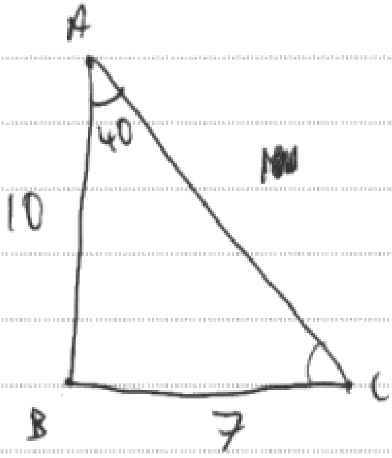
5 In triangle ABC, $AB = 10\text{cm}$, $BC = 7\text{cm}$ and angle $BAC = 40°$ 4

(a) Find, in degrees to the nearest 0.1°, the two possible sizes of angle ACB.

(4)

(b) Find, in cm to 3 significant figures, the difference between the two possible lengths of AC.

(4)



$\frac{7}{\sin(40)} = \frac{10}{\sin(C)}$

a) → $\sin(C) = 0.9$

→ $C =$ ~~[illegible]~~ $66.8, 113.2$

b) $L_1 - L_2$

$AC^2 = 10^2 + 7^2 - 140\cos(73.2)$

⇒ $AC^2 =$ ~~[illegible]~~ $.109$ ⇒ $AC = 10.4$

⇒ $AC^2 = 10^2 + 7^2 - 140\cos(26.8)$

⇒ $AC^2 = 24.0$ ⇒ $AC = 4.90$

→ $10.4 - 4.9 = 5.5$

Response 1

5a. $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\therefore \frac{\sin 40}{7} = \frac{\sin C}{10}$

$\frac{10\sin 40}{7} = \sin C$

$\sin^{-1}\left(\frac{10\sin 40}{7}\right) = C$

$66.7^\circ = ACB$

OR

$\triangle ABC$ could be issocolise

$\therefore \frac{(180-40)}{2} = 70^\circ$

b. $a^2 = b^2 + c^2 - 2bc\cos A$

$\therefore a = \sqrt{(b^2+c^2-2bc\cos A)}$

$\therefore a = \sqrt{(7^2+10^2-2(7)(10)\cos 73.3\ldots)}$

$a = 9.6729\ldots$

$\therefore a = 10\text{cm}$

$\therefore 10 - 9.6729\ldots = 0.327\text{ cm}$